TRINITY COLLEGE

IN NOMINE DOMINI

MATHEMATICS SPECIALIST UNIT 1

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words

Your name

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

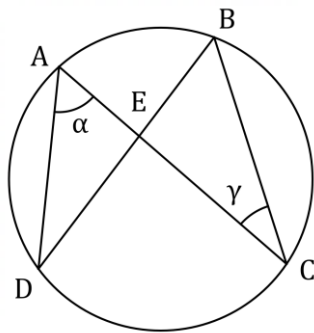**Question 1** **(5 marks)**

(a) In the circle shown, chords AC and BD intersect at E, $\angle ADE = 27°$ and $\angle DEC = 84°$.
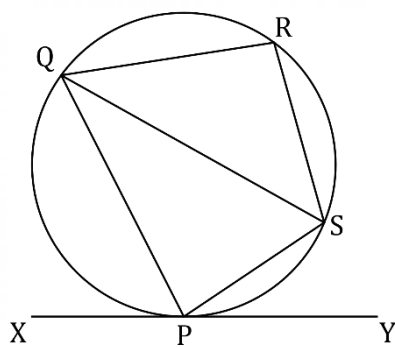
Determine the size of angles α and γ. (2 marks)

(b) In the circle below, $PQRS$ is a cyclic quadrilateral and XY is a tangent to the circle at P.

Given that $\angle XPQ = 72°$ and $\angle PQS = 53°$, determine $\angle QRS$. (3 marks)

Question 2 **(6 marks)**

Points A, B, C and D lie on the circumference of a circle so that the chords AC and BD intersect at point E.

(a) Sketch a diagram to show triangle ADE and triangle BCE and prove that they are similar.

(4 marks)

(b) In the case when the lengths of BE, CE and DE are 7 cm, 4 cm and 6 cm respectively, determine the length of AE. (2 marks)

Question 3 **(6 marks)**

Let $\mathbf{a} = 3\mathbf{i} - 6\mathbf{j}$ and $\mathbf{b} = -\mathbf{i} + \mathbf{j}$.

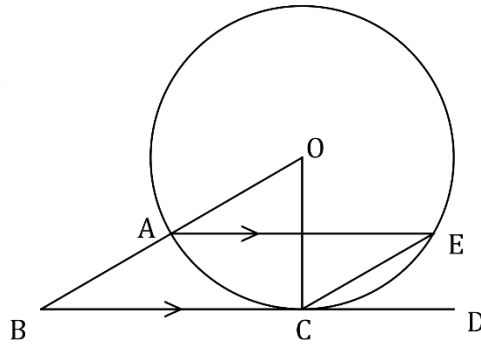(a)   Determine $|2\mathbf{a} + 3\mathbf{b}|$. (3 marks)

(b)   Determine the vectors $\mathbf{m}$ and $\mathbf{n}$ given that $2\mathbf{a} + \mathbf{b} = \mathbf{m} - \mathbf{n}$ and $2\mathbf{b} - \mathbf{a} = 2\mathbf{m} + \mathbf{n}$.

(3 marks)

Question 4 **(4 marks)**

In the diagram, BD is a tangent at C to the circle with centre O, OB intersects the circle at A and chord AE is parallel to tangent BD.

Determine the size of $\angle ECD$ when the size of $\angle ABC = 38°$.

Justify your answer.

See next page

Question 5 **(8 marks)**

(a) Determine the vector(s) that are parallel to $7\mathbf{i} + \mathbf{j}$ and have the same magnitude as $4\mathbf{i} - 3\mathbf{j}$. (4 marks)
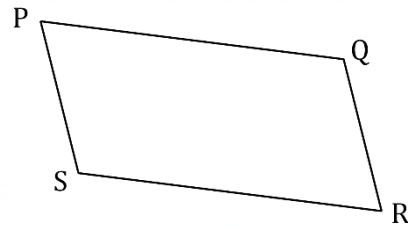
(b) Two vectors are $2\mathbf{i} + (2\lambda + 1)\mathbf{j}$ and $-7\mathbf{i} + (\lambda - 1)\mathbf{j}$, where λ is a constant.

Determine the value(s) of λ so that the vectors are perpendicular. (4 marks)

See next page

Question 6 **(6 marks)**

The diagram shows parallelogram $PQRS$.

A theorem states that the sum of the squares of
the lengths of the diagonals of a parallelogram
is equal to the sum of the squares of the lengths
of its sides.

(a)   Complete the following expression of the theorem using vector notation:

(1 mark)

$$\left|\overrightarrow{PR}\right|^2 + \left|\overrightarrow{QS}\right|^2 =$$

(b)   Letting $\overrightarrow{PQ} = \mathbf{q}$ and $\overrightarrow{PS} = \mathbf{s}$, use a vector method to prove the theorem.   (5 marks)

Question 7 **(7 marks)**

Points A, B and C have position vectors $(-5, -4)$, $(7, 0)$ and $(-2, 7)$ respectively.

(a) Determine the position vector of M, the midpoint of A and B. (1 mark)

(b) Determine the vectors $\overrightarrow{AB}$ and $\overrightarrow{CM}$. (1 mark)

(c) Show that $\overrightarrow{AB}$ and $\overrightarrow{CM}$ are perpendicular. (2 marks)

(d) Hence, or otherwise, determine the area of triangle ABC. (3 marks)

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Question 8   **(8 marks)**

(a)   Show that $3\binom{5}{2} = 5\binom{4}{2}$.   (2 marks)

(b)   Show that $(n-k)\binom{n}{k} = n\binom{n-1}{k}$ for all n and k that are positive integers, $n > k$.

           (4 marks)

(c)   Hence, or otherwise, evaluate $\binom{60}{55}$, given that $\binom{59}{55} = 455\,126$.   (2 marks)

End of questions

Supplementary page

Question number: _____